Factorial ANOVA

- Previously we considered ANOVA applied to 1 independent variable with more than 2 levels/groups
- Factorial ANOVA used in situations with at least 2 independent variables, or factors, each with at least 2 levels
  - each level of 1 factor combined with each level of other factor
  - experimental conditions defined by combinations of factors
- Study with n factors referred to as “n-way factorial design”
  - 2 factors: two-way factorial design
  - if 1 factor has 2 levels and other has 3 levels, often refer to study as using a 2 x 3 factorial design

Eysenck (1974)

- Experiment compared recall memory in younger and older adults in several study conditions
- Independent Variables:
  - Age: younger vs. older
  - Study Condition (Levels of Processing):
    - counting, rhyming, adjective, imagery, intentional
- 2 (Age) x 5 (Study) Factorial Design
  - Ss in each age group assigned randomly to study condition

Concepts from previous lectures

- SS<sub>total</sub>, SS<sub>Group</sub>, SS<sub>error</sub>
- degrees of freedom
- MS<sub>Group</sub>, MS<sub>error</sub>
- F statistic and F distribution
- Association Strength & Effect Size
- Multiple Comparisons
### Conditions in Eysenck (1974)

Balanced factorial design (equal n per cell)

<table>
<thead>
<tr>
<th>Age</th>
<th>Counting</th>
<th>Rhyming</th>
<th>Adjective</th>
<th>Imagery</th>
<th>Intentional</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Younger</td>
<td>n=10</td>
<td>n=10</td>
<td>n=10</td>
<td>n=10</td>
<td>n=10</td>
<td>nc=50</td>
</tr>
<tr>
<td>Older</td>
<td>n=10</td>
<td>n=10</td>
<td>n=10</td>
<td>n=10</td>
<td>n=10</td>
<td>nc=50</td>
</tr>
<tr>
<td>Totals</td>
<td>na=20</td>
<td>na=20</td>
<td>na=20</td>
<td>na=20</td>
<td>na=20</td>
<td>N=nac=100</td>
</tr>
</tbody>
</table>

\*a = number of age groups; c = number of study conditions

### Eysenck Data Table 17.3

- Factorial ANOVA splits total variation in memory scores \(SS_{total}\) into 3 distinct parts:
  - Two Main Effects:
    - Variation due to Age
    - Variation due to Condition
  - One Interaction:
    - Variation due to combinations of Age & Condition
    - does effect of Age depend on Study Condition?
    - does effect of Study Condition depend on Age?

SS\(_{total}\) is measure of total variation of scores

\[
SS_{Total} = \sum (X_{ij} - \bar{X}_{gm})^2
\]

sum taken for all scores in all cells \(ij\)

### SS\(_{Total}\) is measure of total variation of scores

\[SS_{Total} = \sum (X_{ij} - \bar{X}_{gm})^2\]

sum taken for all scores in all cells \(ij\)
### Main Effect of Age

- **SS\textsubscript{Age}:** variation associated with marginal row (age) means
- **sum of deviations of marginal means around grand mean**

\[
SS_{\text{Age}} = n \times c \times \left[ \frac{\sum (X_{Ai} - \bar{X}_{gm})^2}{n} \right]
\]

Sum taken across i=2 Ages

\[n = Ss \text{ per cell} \]
\[c = \text{levels in Condition factor} \]
\[nc = \text{observations per row} \]

### Study Conditions

<table>
<thead>
<tr>
<th>Age</th>
<th>Counting</th>
<th>Writing</th>
<th>Adjective</th>
<th>Imagery</th>
<th>Intermodal</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Old</td>
<td>9</td>
<td>7</td>
<td>11</td>
<td>12</td>
<td>10</td>
<td>10.06</td>
</tr>
<tr>
<td>Young</td>
<td>6</td>
<td>7</td>
<td>11</td>
<td>14</td>
<td>10</td>
<td>13.16</td>
</tr>
</tbody>
</table>

**Note:**
- **SS\textsubscript{Cells}**: sum of deviations of cell means around grand mean
- **SS\textsubscript{error}**: variation that remains after accounting for 2 main effects and interaction:

\[
SS_{\text{error}} = SS_{\text{total}} - (SS_{\text{A}} + SS_{\text{B}} + SS_{\text{AC}})
\]

### Main Effect of Condition

- **SS\textsubscript{Condition}:** variation associated with marginal column (condition) means
- **sum of deviations of marginal means around grand mean**

\[
SS_{\text{Condition}} = n \times a \times \left[ \frac{\sum (X_{Cj} - \bar{X}_{gm})^2}{n} \right]
\]

Sum taken across j=5 Conditions

\[n = Ss \text{ per cell} \]
\[a = \text{levels in Age factor} \]
\[na = \text{observations per column} \]

### Study Conditions

<table>
<thead>
<tr>
<th>Age</th>
<th>Counting</th>
<th>Writing</th>
<th>Adjective</th>
<th>Imagery</th>
<th>Intermodal</th>
<th>Mean</th>
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</thead>
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<td>9</td>
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<tr>
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<td>6</td>
<td>7</td>
<td>11</td>
<td>14</td>
<td>10</td>
<td>13.16</td>
</tr>
</tbody>
</table>

**Note:**
- **SS\textsubscript{Cells}**: sum of deviations of cell means around grand mean
- **SS\textsubscript{error}**: measure of total variation within cells

**Sum taken across all i x j cells**

\[
SS_{\text{error}} = SS_{\text{total}} - (SS_{\text{A}} + SS_{\text{B}} + SS_{\text{AC}})
\]
degrees of freedom

- \( df_{Age} = a - 1 = 2 \)
- \( df_{Condition} = c - 1 = 4 \)
- \( df_{AC} = df_{Age} \times df_{Condition} = 8 \)
- \( df_{Error} = ac(n-1) = 90 \)
- \( df_{Total} = N-1 = nac - 1 = 99 \)

Mean Squares

- \( MS_{Age} = SS_{Age}/df_{Age} \)
- \( MS_{Condition} = SS_{Condition}/df_{Condition} \)
- \( MS_{AC} = SS_{AC}/df_{AC} \)
- \( MS_{Error} = SS_{Error}/df_{Error} \) [equals average of within-cell variances]

When Null Hypothesis is True, each MS is a separate estimate of the population error variance.

Summary ANOVA Table 17.3

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (Age)</td>
<td>1</td>
<td>240.25</td>
<td>240.25</td>
<td>29.94**</td>
<td>.0000</td>
</tr>
<tr>
<td>C (Condition)</td>
<td>4</td>
<td>1514.94</td>
<td>378.735</td>
<td>47.19**</td>
<td>.0000</td>
</tr>
<tr>
<td>AC</td>
<td>4</td>
<td>190.30</td>
<td>47.575</td>
<td>5.93**</td>
<td>.0003</td>
</tr>
<tr>
<td>Error</td>
<td>90</td>
<td>722.30</td>
<td>8.026</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>99</td>
<td>2667.79</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* \( p < .05 \), ** \( p < .01 \)

H0 for Age: The means of the age groups (ignoring condition) do not differ.
H0 for Condition: The means in the various study conditions (ignoring age) do not differ.
H0 for Age \( \times \) Condition: The effect of Age does not depend on Condition (and vice versa).
Main Effect of Age

- Marginal mean of recall for each age group
  - within each age group, plot mean of 5 conditions
- $F(1,90) = 29.94, p < .05$
- Reject null hypothesis that mean recall is the same in each age group

Main Effect of Condition

- Marginal mean of recall for each condition
  - within each condition, plot mean of 2 age groups
- $F(4,90) = 47.19, p < .05$
- Reject null hypothesis that mean recall is the same in all conditions

Age x Condition Interaction

- Difference between ages in each condition
  - $F(4,90) = 5.84, p < .05$
- Reject null hypothesis that effect of age is the same in all conditions
- **Simple main effect** of condition is significant in both age groups but larger in younger Ss
  - younger: $F(4,90)=42, p<.001$
  - older: $F(4,90)=11, p<.001$
- **Simple main effect** of age is not significant in counting & rhyming conditions:
  - intentional: $F(1,90)=33, p<.002$
  - imagery: $F(1,90)=11, p=0.001$
  - adjective: $F(1,90)=9, p=0.003$
  - rhyming: $F(1,90)=0.3, p=0.45$
  - counting: $F(1,90)=0.16, p=0.50$

Interpreting Interactions
Interactions

- 2-way interactions assess whether the effect of one factor depends on the level of the other factor
  - A x B: Does the effect of A depend on the level of B?
    - Does the effect of B depend on the level of A?
- 3-way interactions assess whether the interaction between 2 factors depends on the level of the 3rd factor
  - A x B x C: Does the AxB interaction depend on the level of C?
    - Does the AxC interaction depend on the level of B?
    - Does the BxC interaction depend on the level of A?

Graphical representation of 2-way interactions

- Difference between a1 & a2 depends on level of B.
- Difference between b1 & b2 depends on level of A.

Significant 2-way interactions imply a significant deviation from parallelism.

Graphical representation of 3-way interactions

- Significant AxB interaction
- Non-significant AxBxC interaction

Significant 3-way interactions imply that the deviation from parallelism in a 2-way interaction depends on the level of the 3rd factor

- Significant AxBxC interaction
- AxB interaction depends on level of C
- Significant AxB interaction in c2 but not c1

Incorrect Interpretations of Interactions

- Interaction determines if difference between 2 differences is significant
  - Is \((b_2-b_1)\) at \(a_1\) minus \((b_2-b_1)\) at \(a_2\) significantly different from zero?
- Not the same as doing separate tests of \((b_2-b_1)\) at \(a_1\) and \((b_2-b_1)\) at \(a_2\)
- Example: if t-test 1 is not significant but t-test 2 is significant, will AxB interaction be significant?
  - not necessarily...
Incorrect Interpretations of Interactions

• Suppose both tests are significant... does that mean that the effect of B does not depend on A?
  • i.e., that the AxB interaction is not significant?
  - not necessarily
Ax B interaction is significant & both t-tests are significant

Incorrect Interpretations of Interactions

- Suppose that both tests are not significant... does that mean that the effect of B does not depend on A?
- i.e., that the AxB interaction is not significant?
  - not necessarily
Incorrect Interpretations of Interactions

• To determine if the effect of one variable depends on another
  - e.g., if the effect of one variable differs between groups or ages or genders
• ...you need to assess the interaction between the 2 variables
• should not rely on significance tests performed separately on the different groups

Measures of Association Strength

• eta-squared ($\eta^2$) is proportion of total variation accounted for by each main effect and interaction
• omega-squared ($\omega^2$) is same as eta-squared, but less biased of population value

\[
\hat{\eta}^2_A = \frac{SS_A}{SS_{total}}
\]

\[
\hat{\eta}^2_C = \frac{SS_C}{SS_{total}}
\]

\[
\hat{\eta}^2_{AC} = \frac{SS_{AC}}{SS_{total}}
\]

Measures of Effect Size

• Cohen’s $d$ expresses difference between 2 group means relative to within-group standard deviation
• Cohen’s $f$ is a measure of “typical” difference between group mean and overall mean relative to within-group standard deviation
  - $f$ typically is calculated from partial $\omega^2$
  - when there are 2 groups, $d = 2 f$

\[
d_{Younger-Older} = \frac{X_{Younger} - X_{Older}}{\sqrt{MS_{error}}}
\]

\[
\hat{f}_{Age} = \sqrt{\frac{\hat{\omega}^2_{partial,Age}}{1 - \hat{\omega}^2_{partial,Age}}}
\]
Association strength & effect size

Guidelines from Cohen (1988)

<table>
<thead>
<tr>
<th>Magnitude of association or effect</th>
<th>Partial Omega-squared</th>
<th>Cohen's $f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>small</td>
<td>0.01</td>
<td>0.1</td>
</tr>
<tr>
<td>medium</td>
<td>0.06</td>
<td>0.25</td>
</tr>
<tr>
<td>large</td>
<td>0.14</td>
<td>0.4</td>
</tr>
</tbody>
</table>

ANOVA table for Eysenck (1974) study:

<table>
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<tr>
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<td></td>
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</tr>
</tbody>
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* $p < .05$, ** $p < .01$

eta-squared ($\eta^2$)
- $\eta^2_A = 240.25/2667.79 = 0.09$
- $\eta^2_C = 1514.94/2667.79 = 0.57$
- $\eta^2_{AC} = 190.30/2667.79 = 0.07$

partial eta-squared ($\eta^2_p$)
- $\eta^2_{pA} = 240.25/(240.25+722.30) = 0.25$
- $\eta^2_{pC} = 1514.94/(1514.94+722.30) = 0.67$
- $\eta^2_{pAC} = 190.30/(190.30+722.30) = 0.21$

Assumptions of Factorial ANOVA

- Assumptions are similar to 1-way ANOVA:
  - data within each cell are distributed normally
  - variance is constant across cells/conditions
  - observations/scores are independent

- Decomposition of SS<sub>total</sub> into independent pieces corresponding to main effects and interactions assumes that design is balanced
  - $SS_{total} = SS_A + SS_C + SS_{AC}$

- When design is not balanced, $SS_{total} \neq SS_A + SS_C + SS_{AC}$
- ANOVA for unbalanced factorial designs is more complicated