Decomposing a 3-way interaction

PSYCH 710

Initialize R

Initialize R by entering the following commands at the prompt. You must type the commands *exactly* as shown.

```
options(contrasts=c("contr.sum","contr.poly") ) # set definition of contrasts
load(file=url("http://psycserv.mcmaster.ca/bennett/psy710/datasets/abc.Rdata") )
closeAllConnections()
```

abc

In class we have only considered factorial experiments that include two factors. Here we will examine how to conduct an ANOVA to analyze an experiment that has three factors. In particular, this example illustrates what how to decompose a significant 3-way interaction into *simple interaction effects* and *simple simple main effects*. If the $A \times B \times C$ interaction, then the first step is to analyze the simple interaction effect of $A \times B$ at each level of C (or $A \times C$ at each level of B…or $B \times C$ at each level of A). Essentially, we are reducing our 3-way ANOVA to a series of 2-way ANOVAs. If a simple interaction effect is significant – say the simple interaction effect of $A \times B$ at c1), then we analyze the simple simple main effect of A at b1 and b2 (at c1), and simple simple main effect of B at a1 and a2 (at c1).

A between-subjects factorial design was used to measure the effects of 2 levels of factor A, 2 levels of factor B, and 3 levels of factor C. The data are stored in the data frame `abc.data`. Analyze the data using ANOVA. If there is a significant interaction, analyze the *simple interaction effects* down to the level of simple main effects or *simple simple main effects*. (There is no need to use linear comparisons to analyze main effects, simple main effects, or simple simple main effects).

```
# inspect means, standard deviations, and n:
with(abc.data,tapply(score,list(B,A,C),mean))
with(abc.data,tapply(score,list(B,A,C),sd))
with(abc.data,tapply(score,list(B,A,C),length))
```

```
# overalL ANOVA:
summary(aov(score~A*B*C,data=abc.data))

##             Df Sum Sq Mean Sq F value  Pr(>F)
## A            1     30      30    0.29  0.5897
## B            1     82      82    0.80  0.3742
## C            2    113      56    0.55  0.5794
## A:B          1    727     727    7.08  0.0093 **
## A:C          2    263     132    1.28  0.2832
## B:C          2   2340    1170   11.39 4.2e-05 ***
```

```
## A:B:C        2   1384     692    6.74  0.0019 **
## Residuals   84   8626     103
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

MS.resid.2 <- 102.69
df.resid.2 <- 84
```

Answer: Note that the 3-way interaction is significant. Therefore, we will now analyze *simple interaction effects* by evaluating the $A \times B$ interaction at each level of C.

```
# # simple AxB interaction at c1
summary(aov(score~A*B,data=subset(abc.data,C=="c1")))

##             Df Sum Sq Mean Sq F value Pr(>F)
## A            1    233     233    2.50 0.1248
## B            1   1106    1106   11.89 0.0018 **
## A:B          1    211     211    2.26 0.1436
## Residuals   28   2606      93
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(F.AxB.c1 <- 210.65 / MS.resid.2)

## [1] 2.051

(p.AxB.c1 <- 1-pf(F.AxB.c1,1,df.resid.2))

## [1] 0.1558

# analyze simple main effects of A and B at c1:
(F.A.c1 <- 233.07 / MS.resid.2)

## [1] 2.27

(F.B.c1 <- 1106.43 / MS.resid.2)

## [1] 10.77

(p.A.c1 <- 1-pf(F.A.c1,1,df.resid.2))

## [1] 0.1357

(p.B.c1 <- 1-pf(F.B.c1,1,df.resid.2))

## [1] 0.001501
```

```
# # simple AxB interaction at c2
summary(aov(score~A*B,data=subset(abc.data,C=="c2")))

##             Df Sum Sq Mean Sq F value Pr(>F)
```

```
## A            1     57      57    0.48 0.4953
## B            1   1033    1033    8.66 0.0065 **
## A:B          1    728     728    6.11 0.0198 *
## Residuals   28   3337     119
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(F.AxB.c2 <- 728.46 / MS.resid.2)

## [1] 7.094

(p.AxB.c2 <- 1-pf(F.AxB.c2,1,df.resid.2))

## [1] 0.00927

# simple AxB interaction is significant:
with(subset(abc.data,C=="c2"),tapply(score,list(A,B),mean))

##        b1    b2
## a1 100.24 102.1
## a2  88.03 108.9
```

Answer: The simple interaction $A \times B$ at c2 is significant, so now we analyze the *simple simple main effects* of A at b1 and b2 (at c2), and of B at a1 and a2 (at c2).

```
# simple simple main effect of A at b1 and c2:
summary(aov(score~A,data=subset(abc.data,B=="b1"&C=="c2")))

##             Df Sum Sq Mean Sq F value Pr(>F)
## A            1    596     596    5.77  0.031 *
## Residuals   14   1446     103
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(F.A.b1.c2 <- 596.24/MS.resid.2 )

## [1] 5.806

(p.A.b1.c2 <- 1-pf(F.A.b1.c2,1,df.resid.2))

## [1] 0.01815

# simple simple main effect of A at b2 and c2:
summary(aov(score~A,data=subset(abc.data,B=="b2"&C=="c2")))

##             Df Sum Sq Mean Sq F value Pr(>F)
## A            1    189     189     1.4   0.26
## Residuals   14   1891     135

(F.A.b2.c2 <- 189.11/MS.resid.2 )

## [1] 1.842
```

```
(p.A.b2.c2 <- 1-pf(F.A.b2.c2,1,df.resid.2))

## [1] 0.1784

# simple simple main effect of B at a1 and c2:
summary(aov(score~B,data=subset(abc.data,A=="a1"&C=="c2")))

##             Df Sum Sq Mean Sq F value Pr(>F)
## B            1     13    13.2    0.12   0.73
## Residuals   14   1526   109.0

(F.B.a1.c2 <- 13.24/MS.resid.2 )

## [1] 0.1289

(p.B.a1.c2 <- 1-pf(F.B.a1.c2,1,df.resid.2))

## [1] 0.7204

# simple simple main effect of B at a2 and c2:
summary(aov(score~B,data=subset(abc.data,A=="a2"&C=="c2")))

##             Df Sum Sq Mean Sq F value Pr(>F)
## B            1   1748    1748    13.5 0.0025 **
## Residuals   14   1811     129
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(F.B.a2.c2 <- 1747.94/MS.resid.2 )

## [1] 17.02

(p.B.a2.c2 <- 1-pf(F.B.a2.c2,1,df.resid.2))

## [1] 8.658e-05
```

Answer: Finally, we analyze the simple interaction effect of $A \times B$ at c3:

```
# # simple AxB interaction at c3
summary(aov(score~A*B,data=subset(abc.data,C=="c3")))

##             Df Sum Sq Mean Sq F value Pr(>F)
## A            1      3       3    0.03 0.8572
## B            1    282     282    2.95 0.0970 .
## A:B          1   1172    1172   12.24 0.0016 **
## Residuals   28   2683      96
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(F.AxB.c3 <- 1172.38 / MS.resid.2)

## [1] 11.42
```

```
(p.AxB.c3 <- 1-pf(F.AxB.c3,1,df.resid.2))

## [1] 0.001106

# the simple AxB interaction at c3 is significant
# analyze simple simple main effects of A at b1 and c3:
summary(aov(score~A,data=subset(abc.data,B=="b1"&C=="c3")))

##             Df Sum Sq Mean Sq F value Pr(>F)
## A            1    527     527    4.15  0.061 .
## Residuals   14   1777     127
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(F.A.b1.c3 <- 526.89/MS.resid.2 )

## [1] 5.131

(p.A.b1.c3 <- 1-pf(F.A.b1.c3,1,df.resid.2))

## [1] 0.02608

# simple simple main effect of A at b2 and c3:
summary(aov(score~A,data=subset(abc.data,B=="b2"&C=="c3")))

##             Df Sum Sq Mean Sq F value Pr(>F)
## A            1    649     649      10 0.0069 **
## Residuals   14    906      65
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(F.A.b2.c3 <- 648.65/MS.resid.2 )

## [1] 6.317

(p.A.b2.c3 <- 1-pf(F.A.b2.c3,1,df.resid.2))

## [1] 0.01387
```