Problem Set #4
Miscellaneous Practice Questions

Initialize R by entering the following commands at the prompt. You must type the commands exactly as shown.

```r
> options(contrasts=c("contr.sum","contr.poly") ) # set definition of contrasts
> q1.data <- read.csv(file=url("http://psycserv.mcmaster.ca/bennett/psy710/p1/q1Data.csv") )
```

An experiment was conducted to assess the effects of four treatments on a dependent variable, \( y \). The experiment measured \( y \) on 32 subjects randomly to the four treatments (\( n = 8 \) per treatment), and the data are stored in the data frame `q1.data`. Use `q1.data` to answer all of the following questions.

1. Calculate the mean and standard deviation of \( y \) for each treatment group.

   \[
   \begin{array}{cccc}
   \text{t1} & \text{t2} & \text{t3} & \text{t4} \\
   86.375 & 101.375 & 93.250 & 100.500 \\
   \end{array}
   \]

   \[
   \begin{array}{cccc}
   \text{t1} & \text{t2} & \text{t3} & \text{t4} \\
   9.840695 & 8.895223 & 11.744300 & 7.559289 \\
   \end{array}
   \]

2. Conduct an analysis of variance to evaluate the effect of `treatment` on \( y \). Record the results of your ANOVA (i.e., write the ANOVA table).

   **Analysis of Variance Table**

   Response: y

<table>
<thead>
<tr>
<th></th>
<th>Df</th>
<th>Sum Sq</th>
<th>Mean Sq</th>
<th>F value</th>
<th>Pr(&gt;F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>treatment</td>
<td>3</td>
<td>1182.2</td>
<td>394.08</td>
<td>4.2485</td>
<td>0.01357 *</td>
</tr>
<tr>
<td>Residuals</td>
<td>28</td>
<td>2597.2</td>
<td>92.76</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
   **Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1**

3. Explain the null and alternative hypotheses that are evaluated by your ANOVA.

   **Answer:** We will discuss this answer in class.

4. Explain what is meant by Type I and Type II errors and how they relate to your analysis.

   **Answer:** We will discuss this answer in class.

5. What does Cohen’s \( f \) represent? Calculate Cohen’s \( f \) for `treatment`.

   ```r
   > ( cohens.f <- sqrt(adj.r.squared / (1-adj.r.squared)) )
   [1] 0.5606852
   ```
Answer: Cohen’s $f$ is calculated from adjusted-R-squared, which can be found in the summary table for an `lm` object. For our data, Cohen’s $f$ is 0.5606. We will discuss what Cohen’s $f$ represents in class.

6. Evaluate a linear comparison between the mean of treatment 1 and the mean of the other three treatments.

**Answer 1:** You could answer this question using several methods. For example, you could use the `linear.comparison` command:

```r
# method 1:
> source(url("http://psycserv.mcmaster.ca/bennett/psy710/Rscripts/linear_contrast_v2.R"))
[1] "loading function linear.comparison"
> w <- c(3, -1, -1, -1)
> y <- q1.data$y;
> g <- q1.data$treatment
> my.contrast <- linear.comparison(y, g, c.weights = w)

[1] "computing linear comparisons assuming equal variances among groups"
[1] "C 1: F=9.314, t=-3.052, p=0.005, psi=-36.000, CI=(-60.514,-11.486), adj.CI= (-60.162,-11.838)"

> my.contrast[[1]]$F
[1] 9.314467
> my.contrast[[1]]$df1
[1] 1
> my.contrast[[1]]$df2
[1] 28
> my.contrast[[1]]$p.2tailed
[1] 0.004937769

**Answer 2:** Or you could use R’s `aov` or `lm` commands. First, you would need to associate the contrast weights with the factor, `g`:

```r
# method 2:
> contrasts(g) <- w

> summary(q1.aov.01,split=list(g=list(myContrast=1,others=2:3)))

        Df Sum Sq Mean Sq F value Pr(>F)
g        3 1182.3  394.1  4.248  0.01357 *
g: myContrast 1  864.0  864.0  9.314  0.00494 **
g: others   2  318.2  159.1  1.715  0.19825
Residuals 28  2597.2   92.8
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Instead of listing the ANOVA table, you could list the regression table for an `lm` object. The first line reports the results of a two-tailed $t$ test on our linear contrast. The contrast is significant ($t(28) = -3.052, p = 0.0049$).
Call:
lm(formula = q1.data$y ~ g)

Residuals:
  Min     1Q Median     3Q    Max
-18.250  -8.312   1.562   6.125  18.750

Coefficients:                 Estimate  Std. Error   t value  Pr(>|t|)
(Intercept)          95.3751     1.7033       56.019 < 2e-16 ***
g1                  -3.0000       0.9833       -3.052   0.00494 **
g2                  -6.2233       3.4052       -1.828   0.07829 .
g3                   1.0270       3.4052       0.302    0.76520

---
Signif. codes:  0 '***'  0.001 '**'  0.01 '*'  0.05 '.'  0.1 ' ' 1

Residual standard error: 9.631 on 28 degrees of freedom
Multiple R-squared: 0.3128,  Adjusted R-squared: 0.2392
F-statistic: 4.248 on 3 and 28 DF,  p-value: 0.01357

**Answer 3:** Finally, you could calculate your contrast directly from the group means:

```r
> m <- with(q1.data, tapply(y, treatment, mean)) # group means
> n <- with(q1.data, tapply(y, treatment, length)) # group n
> MS.w <- 92.76 # from ANOVA table
> psi <- sum(w * m) # value of PSI
> # from Eq 6 in chapter 4 notes:
> SS.contrast <- (psi^2) / sum( (w^2)/n)
> (F <- SS.contrast / MS.w )
[1] 9.31436

> N <- sum(n) # total N
> a <- length(m) # number of groups
> df1 <- 1 # numerator df
> (df2 <- N-a ) # denominator df

[1] 28

> (p.value <- 1 - pf(F,df1,df2) ) # our p value [remember to use 1 - pf ]
[1] 0.004937986

> (p.value <- pf(F,df1,df2,lower.tail=FALSE ) ) # [or set lower.tail to FALSE]
[1] 0.004937986
```

7. Assume that you decided to perform the previous contrast after looking at the data. How would your statistical test change? Perform that statistical test.

**Answer:** I would use the Scheffe method. The observed value of $F = 9.31$ is greater than the critical value of $F_{Scheffe} = 8.84$, and therefore the linear contrast is significant ($\alpha_{FW} = .05$).
8. List a set of weights that could be used to evaluate a linear contrast that is orthogonal to the contrast tested in the previous two questions.

**Answer:** Here are my two sets of weights:

\[
\begin{bmatrix}
1 & 3 & -1 & -1 & -1 \\
0 & -2 & 1 & 1 & 1
\end{bmatrix}
\]

There are equal \( n \) per group:

\[t_1 \ t_2 \ t_3 \ t_4\]
\[8 \ 8 \ 8 \ 8\]

Therefore the two contrasts are orthogonal if the sum of the products of the weights is zero:

\[\text{sum}(w1*w2)\]
\[1 \ 0\]

The sum of the products of the weights is zero, so the contrasts are orthogonal.

9. Evaluate all pairwise differences between groups. Make sure the assumptions of your test are valid.

**Answer:** I want to use the Tukey HSD procedure. That procedure assumes that the scores in each group are independent. This assumption is valid because the experiment assigned subjects to groups randomly. Next, we need to show that the groups have equal \( n \). The question says that each group has 8 subjects, but we could verify that for ourselves using the `tapply` command:

\[
t_1 \ t_2 \ t_3 \ t_4
\]
\[8 \ 8 \ 8 \ 8\]

Next, I will use `bartlett.test` to test the assumption of homogeneity of variance:

**Bartlett test of homogeneity of variances**

\[
data: \ y \ by \ treatment \\
\text{Bartlett's K-squared} = 1.3454, \ df = 3, \ p-value = 0.7184
\]

The Bartlett test was not significant \( (K^2 = 1.34, \ df=3, \ p = 0.718) \), so the homogeneity of variance assumption is reasonable for our data. Next, I will test the normality assumption by applying the `shapiro.test` on the residuals of a full linear model fit to the data:

**Shapiro-Wilk normality test**

\[
data: \ \text{residuals(aov(\text{y - treatment, data = q1.data}))} \\
W = 0.9743, \ p-value = 0.626
\]

The Shapiro-Wilk’s test was not significant \( (W = 0.974, \ p = 0.626) \), so the normality assumption is reasonable for our data. Finally, I will compute the Tukey HSD test:
Tukey multiple comparisons of means
95% family-wise confidence level

Fit: aov(formula = y ~ treatment, data = q1.data)

<table>
<thead>
<tr>
<th>treatment</th>
<th>diff</th>
<th>lwr</th>
<th>upr</th>
<th>p adj</th>
</tr>
</thead>
<tbody>
<tr>
<td>t2-t1</td>
<td>15.000</td>
<td>1.8519885</td>
<td>28.148012</td>
<td>0.0207220</td>
</tr>
<tr>
<td>t3-t1</td>
<td>6.875</td>
<td>-6.2730115</td>
<td>20.023012</td>
<td>0.4932943</td>
</tr>
<tr>
<td>t4-t1</td>
<td>14.125</td>
<td>0.9769885</td>
<td>27.273012</td>
<td>0.0316790</td>
</tr>
<tr>
<td>t3-t2</td>
<td>-8.125</td>
<td>-21.2730115</td>
<td>5.023012</td>
<td>0.3490547</td>
</tr>
<tr>
<td>t4-t2</td>
<td>-0.875</td>
<td>-14.0230115</td>
<td>12.273012</td>
<td>0.9978182</td>
</tr>
<tr>
<td>t4-t3</td>
<td>7.250</td>
<td>-5.8980115</td>
<td>20.398012</td>
<td>0.4477413</td>
</tr>
</tbody>
</table>

Only the differences between treatments 1 and 2 (p_{adj} = 0.0207) and treatments 1 and 4 (p_{adj} = 0.031) were significant.