Launch R, navigate to a suitable working directory, and then enter the following command:

```
> options(contrasts=c("contr.sum","contr.poly")) # sum-to-zero effects
```

For all of the following analyses, unless you are instructed otherwise you may assume that the assumptions of normality and homogeneity of variance are valid.

1. An experiment measured performance of 40 subjects assigned randomly to four different test conditions, with the constraint that there were 10 subjects per group. Each subject was tested only once. The independent (i.e., grouping) variable is `condition`; the dependent variable is `score`.

   (a) Read the data file into R and with the following command:
   ```
   > q1data <- read.csv("http://psycserv.mcmaster.ca/bennett/psy710/problems/ps3/ps3q1data.csv")
   ```

   (b) Calculate the group means and standard deviations.
   ```
   > (theMeans <- with(q1data,tapply(score,condition,mean)) )
   condition1 condition2 condition3 condition4
   95 104 96 105
   > (theSDs <- with(q1data,tapply(score,condition,sd)) )
   condition1 condition2 condition3 condition4
   8.891789 14.716348 8.230862 14.464102
   ```

   (c) Perform a one-way ANOVA to evaluate the effect of `condition`. Write the ANOVA table.
   ```
   > aov.01 <- aov(score~condition,data=q1data)
   > summary(aov.01);
   Df  Sum Sq Mean Sq F value Pr(>F)
   condition  3  492 164.0  1.146  0.355
   Residuals 20 2863 143.2
   ```

   (d) Perform a single linear contrast to evaluate the hypothesis that the means of groups 1 and 3 are lower than the means of groups 2 and 4. Write the value of your statistic (i.e., $F$ or $t$), the degrees of freedom, the p-value, and your conclusion.

   **Answer:** I will set up my contrast weights so that my null hypothesis is $\psi \leq 0$ and the alternative hypothesis is $\psi > 0$. I will evaluate the null hypothesis with a one-tailed $t$ test: To reject the null hypothesis, I need to show that $t$ is significantly greater than zero.
   ```
   > levels(q1data$condition)
   [1] "condition1" "condition2" "condition3" "condition4"
   > with(q1data,tapply(score,condition,length)) # equal n
   condition1 condition2 condition3 condition4
   6  6    6  6
   ```
> c1 <- c(-1,1,-1,1) # contrast weights to test hypothesis
> MS.resid <- 143.2;
> df.resid <- 20
> (psi <- sum(c1*theMeans))
[1] 18
> (group.n <- with(q1data,tapply(score,condition,length)))
 condition1  condition2  condition3  condition4
            6            6            6            6
> # t test:
> (t <- ( psi / sqrt( sum((c1^2)/group.n) ) ) / sqrt(MS.resid) ) # t statistic
[1] 1.842242
> # probability of getting a t at least this big:
> (p.big <- pt(t,df.resid,lower.tail=FALSE) )
[1] 0.04015847
> # reject null hypoth that psi less-than-or-equal to zero
> # using lm():
> contrasts(q1data$condition) <- c1
> lm.01 <- lm(score~condition,data=q1data)
> summary(lm.01)

Call:  
  lm(formula = score ~ condition, data = q1data)

Residuals:  
    Min     1Q Median     3Q    Max  
-22.4500 -6.8702  0.7245  5.8233 18.4666

Coefficients:  
              Estimate Std. Error  t value Pr(>|t|)  
(Intercept)    100.000     2.442  40.946  <2e-16 ***
condition1     4.5000     2.442   1.843   0.0803 .
condition2     0.6000     4.885   0.123   0.9035
condition3     0.8000     4.885   0.164   0.8715

---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 11.96 on 20 degrees of freedom
Multiple R-squared: 0.1466,  Adjusted R-squared: 0.01865
F-statistic: 1.146 on 3 and 20 DF,  p-value: 0.3549
> (p.one.tailed <- .0803/2)
[1] 0.04015

(e) Construct the weights for a complete, orthogonal set of linear comparisons. State the hypothesis being evaluated by each comparison.

(f) Evaluate all pairwise comparisons between groups while maintaining a family-wise Type I error rate of 0.05.

2. An experiment used a between-subjects design: 54 subjects were assigned randomly to 6 conditions, with the constraint that 9 subjects were assigned to each condition. Here are the means, standard deviations, and n for each group:
An ANOVA was performed to evaluate the effect of condition. Here is the ANOVA table:

<table>
<thead>
<tr>
<th></th>
<th>Df</th>
<th>Sum Sq</th>
<th>Mean Sq</th>
<th>F value</th>
<th>Pr(&gt;F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>condition</td>
<td>5</td>
<td>1723</td>
<td>344.7</td>
<td>3.781</td>
<td>0.00575 **</td>
</tr>
<tr>
<td>Residuals</td>
<td>48</td>
<td>4376</td>
<td>91.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(a) After inspecting the data, you decide to conduct two linear contrasts the compare i) the mean of groups 1-3 to the mean of groups 4-6; and ii) the mean of group1 to the mean of groups 2-3. Perform both contrasts, keeping the family-wise Type I error rate at $\alpha_{FW} = 0.05$. For each contrast, show the contrast weights, $F$, and degrees of freedom, and state your conclusion regarding the null and alternative hypotheses.

> # will use Scheffe method
> a <- 6 # 6 groups
> N <- 54 # total N
> alpha.fw <- 0.05 # familywise alpha
> # calculate critical value of F...
> # alpha.fw is probability of exceeding a critical value of F, but
> # the default qf command expects to get the probability of BINGO!!!!!
> # LESS than a critical value of F, so we use 1-alpha.fw instead of alpha.fw:
> ( F.scheffe <- (a-1) * qf(1-alpha.fw,df1=a-1,df2=N-a) )
[1] 12.04257

> the.means <- c(90,95,92,100,102,106)
> group.n <- c(9,9,9,9,9,9)
> MS.resid <- 91.2 # Mean Square Residuals
> df.resid <- 48
> c1 <- c(-1,-1,-1,1,1,1) # weights for contrast 1
> c2 <- c(-1,1/2,1/2,0,0,0); # weights for contrast 2
> # check they sum to zero:
> sum(c1)
[1] 0
> sum(c2)
[1] 0
> c1sqr <- c1^2 # squared weights
> c2sqr <- c2^2 # squared weights
> # do contrast 1:
> psi <- sum(the.means * c1)
> (F.c1 <- ( (psi^2) / sum( c1sqr / group.n) ) / MS.resid )
[1] 15.80592
> # this value of F exceeds F.scheffe, so the contrast is significant
> # and we reject null hypoth of no diff between means grps 1-3 and grps 4-6
> # do contrast 2:
> psi <- sum(the.means * c2)
> (F.c2 <- ( (psi^2) / sum( c2sqr / group.n) ) / MS.resid )
[1] 0.8059211
> # this value of F is less than F.scheffe, so the contrast is not significant
> # fail to reject the null hypoth of no diff between means grps 2-3 and grp 1

(b) Are these contrasts orthogonal? Explain.
> sum(c1*c2)
[1] 0

Answer: The groups all have equal \( n \), and therefore, if the contrasts are orthogonal, the sum of the products of the weights should be zero. The sum of the products is zero, and so the contrasts are orthogonal.

(c) Now assume that the two linear contrasts described in 2a were planned comparisons, and we wanted to maintain \( \alpha_{FW} \leq 0.05 \). Explain how you would control the Type I error rate, taking care to explain how your method would differ from the one used in 2a.
> c <- 2 # number of comparisons
> alpha.fw <- .05
> (p1.critical <- alpha.fw/c )
[1] 0.025
> (p1.observed <- pf(F.c1,1,df.resid,lower.tail=FALSE) )
[1] 0.0002354368
> print("1st contrast is significant; reject null hypothesis")
[1] "1st contrast is significant; reject null hypothesis"
> (p2.critical <- alpha.fw / (c-1) )
[1] 0.05
> (p2.observed <- pf(F.c2,1,df.resid,lower.tail=FALSE) )
[1] 0.3738099
> print("2nd contrast is not significant; do not reject null hypothesis")
[1] "2nd contrast is not significant; do not reject null hypothesis"

Answer: I would use the Holm’s sequential Bonferroni test. First, I would rank-order the two comparisons from highest to lowest value of \( F \). Then I would evaluate the first \( F \) using a critical p-value of \( \alpha_{FW} = 0.05/2 = 0.025 \). If the first \( F \) is significant, then I would evaluate the second \( F \) using a critical p-value of \( \alpha_{FW} = 0.05/(2-1) = 0.05 \). Using this procedure, the first contrast between the means of groups 1-3 and groups 4-6 is significant, but the second one is not.