PSYCH 710

Higher-Order Within-Subjects ANOVA

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Factorial Within-Subject Design

- 2x3 within-subjects factorial design
- A & B are crossed, fixed factors
- subjects is a random factor
- typically, 1 observation per cell
  - not possible to measure within-cell variance
  - cannot distinguish contributions of error and subject x treatment interaction to between-cell variation
- data from each subject are correlated

	A_1B_1	A_1B_2	A_1B_3	A_2B_1	A_2B_2	A_2B_3
subject 1	n=1	n=1	n=1	n=1	n=1	n=1
subject 2	n=1	n=1	n=1	n=1	n=1	n=1
subject 3	n=1	n=1	n=1	n=1	n=1	n=1
subject 4	n=1	n=1	n=1	n=1	n=1	n=1

Full Model and F tests

$$Y_{ijk} = \mu + \alpha_j + \beta_k + \pi_i + (\alpha\beta)_{jk} + (\alpha\pi)_{ji} + (\beta\pi)_{ki} + (\alpha\beta\pi)_{jki} + \epsilon_{ijk}$$

- Statistical significance of all parameters evaluated by comparing $SS_{residuals}$ obtained with nested models
- error terms for F tests slightly more complicated:
  - main effect of A evaluated with A x Subjects term
  - main effect of B evaluated with B x Subjects term
  - A x B interaction evaluated with A x B x Subjects term

Effect	**E**(Mean Square)	F
S	$\sigma_\epsilon^2 + ab\sigma_\pi^2$	
A	$\sigma_\epsilon^2 + b\sigma_{\alpha\pi}^2 + nb\frac{\sum_j \alpha_j^2}{a-1}$	$\frac{MS_A}{MS_{A\times S}}$
$A \times S$	$\sigma_\epsilon^2 + b\sigma_{\alpha\pi}^2$	
B	$\sigma_\epsilon^2 + a\sigma_{\beta\pi}^2 + na\frac{\sum_k \beta_j^2}{b-1}$	$\frac{MS_B}{MS_{B\times S}}$
$B \times S$	$\sigma_\epsilon^2 + a\sigma_{\beta\pi}^2$	
$A \times B$	$\sigma_\epsilon^2 + \sigma_{\alpha\beta\pi}^2 + n\frac{\sum_j \sum_k (\alpha\beta)_{jk}^2}{(a-1)(b-1)}$	$\frac{MS_{A\times B}}{MS_{A\times B\times S}}$
$A \times B \times S$	$\sigma_\epsilon^2 + \sigma_{\alpha\beta\pi}^2$	

R example

- Effects of noise (distractors) and stimulus orientation on letter discrimination
- 2 (noise) x 3 (orientation) within-subject factorial design

```
> load(url("http://pnb.mcmaster.ca/bennett/psy710/datasets/rtData-mw12-1.rda"))
> sapply(rt.wide,class)
     subj  absent.a0  absent.a4  absent.a8 present.a0 present.a4 present.a8
 "factor"  "numeric"  "numeric"  "numeric"  "numeric"  "numeric"  "numeric"
> summary(rt.long)
      subj        noise     angle        rt
 s1     : 6   absent :30   a0:20   Min.   : 284
 s10    : 6   present:30   a4:20   1st Qu.: 459
 s2     : 6                a8:20   Median : 571
 s3     : 6                        Mean   : 588
 s4     : 6                        3rd Qu.: 686
 s5     : 6                        Max.   :1000
 (Other):24
```

R example

```
> options(width=80,digits=4)
> options(contrasts=c("contr.sum","contr.poly"))

> # following model is incorrect
> rt.aov.00 <- aov(rt ~ 1+Error(subj/(angle*noise)),
+                 data=rt.long) # no fixed effects!
```

R example

```
> options(width=80,digits=4)
> options(contrasts=c("contr.sum","contr.poly"))

> # the next 2 models are equivalent:
> rt.aov.01 <- aov(rt ~ angle*noise+Error(subj/(angle*noise)),
+               data=rt.long)

> # rt.aov.01b <- aov(rt ~ angle*noise,
+                  + Error(subj/(angle+noise+angle:noise)),
+                  data=rt.long)
```

R example

```
> rt.aov.01 <- aov(rt ~ angle*noise+Error(subj/(angle*noise)),data=rt.long)
> summary(rt.aov.01)
Error: subj
          Df Sum Sq Mean Sq F value Pr(>F)
Residuals  9 344423   38269

Error: subj:angle
          Df Sum Sq Mean Sq F value Pr(>F)
angle      2 225422  112711    7.79 0.0036 **
Residuals 18 260280   14460
Error: subj:noise
          Df Sum Sq Mean Sq F value Pr(>F)
noise      1 234625  234625    15.7 0.0033 **
Residuals  9 134551   14950
Error: subj:angle:noise
            Df Sum Sq Mean Sq F value Pr(>F)
angle:noise  2 187983   93992    7.51 0.0043 **
Residuals   18 225402   12522
```

assumes sphericity (subj:angle)

assumes sphericity (subj:angle:noise)

R example (aov_car)

```
> library(afex)
> rt.aov.02 <-aov_car(rt ~ angle*noise+Error(subj/(angle*noise)),data=rt.long)
> summary(rt.aov.02)
Univariate Type III Repeated-Measures ANOVA Assuming Sphericity
              Sum Sq num Df Error SS den Df F value  Pr(>F)
(Intercept) 20765813      1   344423      9  542.62 2.4e-09 ***
angle         225423      2   260280     18    7.79  0.0036 **
noise         234625      1   134551      9   15.69  0.0033 **
angle:noise   187983      2   225402     18    7.51  0.0043 **
```

R example (aov_car)

```
> rt.aov.02 <-aov_car(rt ~ angle*noise+Error(subj/(angle*noise)),data=rt.long)
> summary(rt.aov.02)
Univariate Type III Repeated-Measures ANOVA Assuming Sphericity
              Sum Sq num Df Error SS den Df F value  Pr(>F)
(Intercept) 20765813      1   344423      9  542.62 2.4e-09 ***
angle         225423      2   260280     18    7.79  0.0036 **
noise         234625      1   134551      9   15.69  0.0033 **
angle:noise   187983      2   225402     18    7.51  0.0043 **
```

```
Mauchly Tests for Sphericity
            Test statistic p-value
angle                0.690   0.227
angle:noise          0.639   0.167
```

```
Greenhouse-Geisser and Huynh-Feldt Corrections
            GG eps Pr(>F[GG])
angle        0.764     0.0083 **
angle:noise  0.735     0.0103 *
            HF eps Pr(>F[HF])
angle       0.8880   0.005369
angle:noise 0.8431   0.007177
```

Strength of Association & Effect Size

omega-squared

$$\hat{\omega}^2 = \frac{df_{\,effect}(MS_{\,effect} - MS_{\,effect \times S})}{SS_{\,effect} + SS_{\,effect \times S} + SS_{\,S} + MS_{\,S}}$$

Cohen's f

$$\hat{f} = \sqrt{\frac{\hat{\omega}^2_{effect}}{1 - \hat{\omega}^2_{effect}}}$$

Strength of Association & Effect Size

```
library(effectsize)
omega_squared(rt.aov.02)

## # Effect Size for ANOVA (Type III)
##
## Parameter   | Omega2 (partial) |       95% CI
## ---------------------------------------------
## angle       |             0.23 | [0.00, 1.00]
## noise       |             0.29 | [0.00, 1.00]
## angle:noise |             0.20 | [0.00, 1.00]
##
## - One-sided CIs: upper bound fixed at (1).
```

Strength of Association & Effect Size

```
eta_squared(rt.aov.02)

## # Effect Size for ANOVA (Type III)
##
## Parameter   | Eta2 (partial) |       95% CI
## ---------------------------------------------
## angle       |           0.46 | [0.14, 1.00]
## noise       |           0.64 | [0.23, 1.00]
## angle:noise |           0.45 | [0.13, 1.00]
##
## - One-sided CIs: upper bound fixed at (1).
```

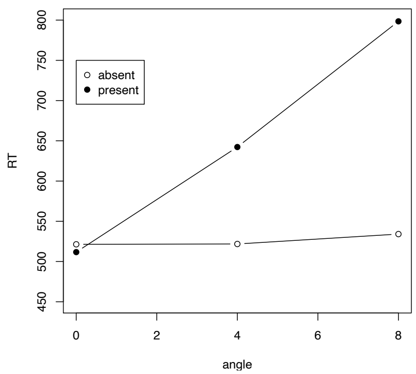
Strength of Association & Effect Size

```
cohens_f(rt.aov.02)

## # Effect Size for ANOVA (Type III)
##
## Parameter   | Cohen's f (partial) |          95% CI
## ---------------------------------------------------
## angle       |                0.93 | [0.41,      Inf]
## noise       |                1.32 | [0.55,      Inf]
## angle:noise |                0.91 | [0.39,      Inf]
##
## - One-sided CIs: upper bound fixed at (Inf).
```

simple main effects

- use simple main effects to decompose an interaction
- unlike between-subjects designs, it is better to use <u>separate error terms</u> for each simple main effect
  - do not recalculate F with MS_{error} and df_{error} from main analysis
- each simple main effect is a 1-way within-subject ANOVA



simple main effects

```
aov.angle.absent <- aov_car(rt~angle+Error(subj/angle),data=subset(rt.long,noise=="absent"))
aov.angle.present <- aov_car(rt~angle+Error(subj/angle),data=subset(rt.long,noise=="present"))
# Hyunh-Feldt correction; pes == partial eta squared:
anova(aov.angle.absent,correction="HF",es="pes")


## Anova Table (Type 3 tests)
##
## Response: rt
##       num Df den Df   MSE    F     pes Pr(>F)
## angle      2     18 10460 0.05 0.00551   0.95


anova(aov.angle.present,correction="HF",es="pes")


## Anova Table (Type 3 tests)
##
## Response: rt
##       num Df den Df   MSE    F   pes Pr(>F)
## angle      2     18 16522 12.5 0.581  4e-04 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

linear contrasts

- similar to procedures used with 1-way within-subjects ANOVA
- use contrast weights to create composite scores
  - converts multivariate analysis to univariate analysis
- use t test to evaluate null hypothesis

linear contrasts example

evaluate linear trend of RT across angle on entire data set (i.e., ignoring noise)

```
rt.mat <- as.matrix(rt.wide[,2:7])
rt.mat[1:2,]

##   absent.a0 absent.a4 absent.a8 present.a0 present.a4 present.a8
## 1       657       446       461        671        474        903
## 2       450       448       484        284        562        585

lin.C <- c(-1,0,1,-1,0,1)
rt.lin <- rt.mat %*% lin.C
```

linear contrasts example

evaluate linear trend of RT across angle on entire data set (i.e., ignoring noise)

```
lin.C <- c(-1,0,1,-1,0,1)
rt.lin <- rt.mat %*% lin.C
t.test(rt.lin)

##
##      One Sample t-test
##
## data:  rt.lin
## t = 3.5, df = 9, p-value = 0.007
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  104.9 494.1
## sample estimates:
## mean of x
##     299.5
```

linear contrasts example

evaluate linear trend of RT across angle <u>separately at each level of noise</u>

```
rt.mat[1,] # inspect column names

##  absent.a0  absent.a4  absent.a8 present.a0 present.a4 present.a8
##        657        446        461        671        474        903

# note the contrast weights in next 2 lines:
rt.absent.lin <- rt.mat %*% c(-1,0,1,0,0,0)
rt.present.lin <- rt.mat %*% c(0,0,0,-1,0,1)
```

linear contrasts example

evaluate linear trend of RT across angle separately at each level of noise

```
# note the contrast weights in next 2 lines:
rt.absent.lin <- rt.mat %*% c(-1,0,1,0,0,0)
rt.present.lin <- rt.mat %*% c(0,0,0,-1,0,1)
t.test(rt.absent.lin)

##
##      One Sample t-test
##
## data:  rt.absent.lin
## t = 0.32, df = 9, p-value = 0.8
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  -77.93 103.33
## sample estimates:
## mean of x
##      12.7
```

linear contrasts example

evaluate linear trend of RT across angle separately at each level of noise

```
# note the contrast weights in next 2 lines:
rt.absent.lin <- rt.mat %*% c(-1,0,1,0,0,0)
rt.present.lin <- rt.mat %*% c(0,0,0,-1,0,1)

t.test(rt.present.lin)

##
##      One Sample t-test
##
## data:  rt.present.lin
## t = 4.8, df = 9, p-value = 0.001
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  151.7 421.9
## sample estimates:
## mean of x
##     286.8
```

linear contrasts example

does linear trend of RT across angle differ across noise levels?

weights = $\Psi(-1,0,1,0,0,0) - \Psi(0,0,0,-1,0,1) = \Psi(-1,0,1,1,0,-1)$

```
myC <- c(-1,0,1,1,0,-1)
rt.lin.x.noise <- rt.mat %*% myC
t.test(rt.lin.x.noise)
```

contrast weights (linear trend x noise interaction)

composite scores

2-tailed t test

```
##
##      One Sample t-test
##
## data:  rt.lin.x.noise
## t = -5.1, df = 9, p-value = 7e-04
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  -396.8 -151.4
## sample estimates:
## mean of x
##    -274.1
```

noise x linear trend interaction is significant

linear contrasts example

using ANOVA to evaluate linear trend x noise interaction

```
N <- length(rt.absent.lin) # number of subjects
linTrend <- c(rt.pres.lin,rt.absent.lin)
nz <- as.factor(x=rep(c("present","absent"),each=N))
sid <- factor(x=rep(1:N,times=2),label="s")
linTrend.df <- data.frame(sid,nz,linTrend)
summary(linTrend.df)

##       sid           nz         linTrend
##  s1     :2   absent :10   Min.   :-196
##  s2     :2   present:10   1st Qu.:  19
##  s3     :2                Median : 104
##  s4     :2                Mean   : 150
##  s5     :2                3rd Qu.: 249
##  s6     :2                Max.   : 575
##  (Other):8
```

create data frame of composite/trend scores for noise present & absent conditions

linear contrasts example

using ANOVA to evaluate linear trend x noise interaction

evaluate the effect of noise on composite scores with 1-way within-subjects ANOVA

```
options(contrasts=c("contr.sum","contr.poly"))
linTrend.aov.01 <- aov_car(linTrend~nz+Error(sid/nz),data=linTrend.df)
summary(linTrend.aov.01)

##
## Univariate Type III Repeated-Measures ANOVA Assuming Sphericity
##
##             Sum Sq num Df Error SS den Df F value   Pr(>F)
## (Intercept) 448501      1   333077      9    12.1 0.00693 **
## nz          375654      1   132496      9    25.5 0.00069 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

what do these effects mean?

linear contrasts (emmeans)

evaluate linear trend of RT across angle on entire data set (i.e., ignoring noise)

```
library(emmeans)
rt.emm <- emmeans(rt.aov.02,specs="angle")
lin.C <- c(-1,0,1) # trend weights

 # linear trend ignoring noise
contrast(rt.emm,method=list(linear=lin.C))

##  contrast estimate SE df t.ratio p.value
##  linear        150 43  9   3.481  0.0069
##
## Results are averaged over the levels of: noise
```

linear contrasts (emmeans)

evaluate linear trend of RT across angle separately at each level of noise

```
# linear trend separately for each noise
rt.emm.2 <- emmeans(rt.aov.02,specs="angle",by="noise")
contrast(rt.emm.2,method=list(linear=lin.C))

## noise = absent:
##  contrast estimate   SE df t.ratio p.value
##  linear       12.7 40.1  9   0.317  0.7585
##
## noise = present:
##  contrast estimate   SE df t.ratio p.value
##  linear      286.8 59.7  9   4.801  0.0010
```

linear contrasts (emmeans)

linear trend x noise interaction

```
# linear trend x noise interaction:
contrast(rt.emm.2,interaction=list(angle=list(lin.C),noise=list(c(-1,1))),by=NULL)

##  angle_custom noise_custom estimate   SE df t.ratio p.value
##  c(-1, 0, 1)  c(-1, 1)          274 54.3  9   5.051  0.0007
```

alternative methods (lmer)

```
# using lmer:
library(lmerTest)
rt.lmer.01 <- lmer(rt~angle*noise+(1|subj),data=rt.long)
anova(rt.lmer.01) # assumes sphericity

## Type III Analysis of Variance Table with Satterthwaite's method
##             Sum Sq Mean Sq NumDF DenDF F value   Pr(>F)
## angle       225422  112711     2    45    8.18 0.00093 ***
## noise       234575  234575     1    45   17.02 0.00016 ***
## angle:noise 187983   93992     2    45    6.82 0.00259 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

assumes sphericity

alternative methods (lmer)

conservative F test

```
> # conservative F tests:
> N <- 10 # 10 subjects
> F.angle <- 8.18
> F.angle.x.noise <- 6.82
> # conservative F tests for angle and angle x noise:
> 1-pf(F.angle,1,N) # p value for angle
[1] 0.01695
> 1-pf(F.angle.x.noise,1,N) # p value for angle x noise
[1] 0.02597
```

alternative methods (lmer)

chi-square tests

```
> library(car)
> Anova(rt.lmer.01,type="III")
Analysis of Deviance Table (Type III Wald chisquare tests)

Response: rt
            Chisq Df Pr(>Chisq)
(Intercept) 542.6  1    < 2e-16 ***
angle        16.4  2    0.00028 ***
noise        17.0  1    3.7e-05 ***
angle:noise  13.6  2    0.00109 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

association strength (lmer)

fixed effects

```
library(effectsize)
omega_squared(rt.lmer.01)

## # Effect Size for ANOVA (Type III)
##
## Parameter   | Omega2 (partial) |       95% CI
## ---------------------------------------------
## angle       |             0.23 | [0.06, 1.00]
## noise       |             0.25 | [0.09, 1.00]
## angle:noise |             0.20 | [0.04, 1.00]
##
## - One-sided CIs: upper bound fixed at (1).
```

effect size (lmer)

fixed effects

```
cohens_f(rt.lmer.01)

## # Effect Size for ANOVA (Type III)
##
## Parameter   | Cohen's f (partial) |          95% CI
## -------------------------------------------------------
## angle       |                0.60 | [0.31,       Inf]
## noise       |                0.61 | [0.34,       Inf]
## angle:noise |                0.55 | [0.25,       Inf]
##
## - One-sided CIs: upper bound fixed at (Inf).
```

variance components (lmer)

```
> # random components
> ranova(rt.lmer.01) # anova-like table

ANOVA-like table for random-effects: Single term deletions

Model:
rt ~ angle + noise + (1 | subj) + angle:noise
           npar logLik AIC   LRT Df Pr(>Chisq)
<none>        8   -350 715
(1 | subj)    7   -352 718 4.81  1      0.028 *
```

variance components (lmer)

```
> # variance components
> print(VarCorr(rt.lmer.01),comp=c("Variance","Std.Dev."))
 Groups   Name        Variance Std.Dev.
 subj     (Intercept)  4081     63.9
 Residual             13783    117.4

> # association strength
> library(performance)
> icc(rt.lmer.01,by_group = T)
# ICC by Group
Group |   ICC
-------------
subj  | 0.228
```

alternative methods (lme in nlme)

models differ in assumed correlation structure in residuals

```
> # using lme in nlme
> library(nlme)
> # assumes independence:
> rt.lme.01 <- lme(rt~angle*noise,random=~1|subj,
+                  data=rt.long)
> # assumes sphericity:
> rt.lme.02 <- lme(rt~angle*noise,random=~1|subj,
+                  data=rt.long,
+                  correlation=corCompSymm(value=0.3,form=~1|subj))
> # does not assume sphericity:
> rt.lme.03 <- lme(rt~angle*noise,random=~1|subj,
+                  data=rt.long,
+                  correlation=corSymm(form=~1|subj))
```

alternative methods (lme in nlme)

models differ in assumed correlation structure in residuals

```
> # no significant difference in fit:
> anova(rt.lme.01,rt.lme.02,rt.lme.03)
          Model df   AIC   BIC logLik   Test L.Ratio p-value
rt.lme.01     1  8 715.5 731.4 -349.7
rt.lme.02     2  9 717.5 735.4 -349.7 1 vs 2    0.00  0.9999
rt.lme.03     3 23 728.3 774.0 -341.1 2 vs 3   17.23  0.2444
```

alternative methods (lme in nlme)

fixed effects ANOVA

```
# fixed effects for model 1
anova(rt.lme.01)

##              numDF denDF F-value p-value
## (Intercept)      1    45   542.6  <.0001
## angle            2    45     8.2  0.0009
## noise            1    45    17.0  0.0002
## angle:noise      2    45     6.8  0.0026
```

alternative methods (lme in nlme)

fixed effects association strength

```
omega_squared(rt.lme.01)

## # Effect Size for ANOVA
##
## Parameter   | Omega2 (partial) |       95% CI
## ---------------------------------------------
## angle       |             0.23 | [0.06, 1.00]
## noise       |             0.25 | [0.09, 1.00]
## angle:noise |             0.20 | [0.04, 1.00]
##
## - One-sided CIs: upper bound fixed at (1).
```

alternative methods (lme in nlme)

fixed effects effect size

```
cohens_f(rt.lme.01)

## # Effect Size for ANOVA
##
## Parameter   | Cohen's f (partial) |         95% CI
## -------------------------------------------------
## angle       |                0.60 | [0.31,    Inf]
## noise       |                0.62 | [0.34,    Inf]
## angle:noise |                0.55 | [0.25,    Inf]
##
## - One-sided CIs: upper bound fixed at (Inf).
```

variance components (lme in nlme)

model parameters fit with assumption that residuals were independent

```
# variance components (independence)
print(VarCorr(rt.lme.01),comp=c("Variance","Std.Dev."))
## subj = pdLogChol(1)
##             Variance StdDev
## (Intercept)  4081     63.89
## Residual    13783    117.40


( icc.subj <- 4081/(4081+13783) )

## [1] 0.2284
```

variance components (lme in nlme)

model parameters fit with compound symmetric variance-covariance matrix for residuals

```
# variance components (compound symmetry)
print(VarCorr(rt.lme.02),comp=c("Variance","Std.Dev."))

## subj = pdLogChol(1)
##             Variance StdDev
## (Intercept)  1142     33.79
## Residual    16722    129.31


( icc.subj <- 1142/(1142+16722) )

## [1] 0.06393
```

split-plot (between-within, mixed) designs

- split-plot designs have between-subject & within-subject factors
- analyzed the same way as within-subjects design except we include between-subjects factors in multivariate linear model

split-plot designs

```
> library(afex)
> options(contrasts=c("contr.sum","contr.poly"))
> rtAge.aov.02 <- aov_car(rt ~ group*angle + Error(subj/angle),data=rtVisual)
> summary(rtAge.aov.02)

Univariate Type III Repeated-Measures ANOVA Assuming Sphericity

             Sum Sq num Df Error SS den Df F value   Pr(>F)
(Intercept) 9610000      1   100222     10  958.87 2.9e-11 ***
group         16044      1   100222     10    1.60   0.234
angle         28850      2   114311     20    2.52   0.105
group:angle   50772      2   114311     20    4.44   0.025 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

split-plot designs

```
            Sum Sq num Df Error SS den Df F value  Pr(>F)    
(Intercept) 9610000      1   100222     10  958.87 2.9e-11 ***
group         16044      1   100222     10    1.60   0.234    
angle         28850      2   114311     20    2.52   0.105    
group:angle   50772      2   114311     20    4.44   0.025 *  

Mauchly Tests for Sphericity
            Test statistic p-value
angle                0.354 0.00933
group:angle          0.354 0.00933

Greenhouse-Geisser & Huynh-Feldt Corrections
             GG eps Pr(>F[GG])
angle        0.607      0.135
group:angle  0.607      0.051 .
            HF eps Pr(>F[HF])
angle       0.6468    0.13155
group:angle 0.6468    0.04726
```

sphericity assumption applies to all effects that include a within-subjects factor (i.e., even between x within interactions)

split-plot designs

between-group portion is equivalent to 1-way ANOVA on average scores for each S

```
##
## Univariate Type III Repeated-Measures ANOVA Assuming Sphericity
##
##              Sum Sq num Df Error SS den Df F value  Pr(>F)    
## (Intercept) 9610000      1   100222     10  958.87 2.9e-11 ***
## group         16044      1   100222     10    1.60   0.234    
## angle         28850      2   114311     20    2.52   0.105    
## group:angle   50772      2   114311     20    4.44   0.025 *  
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
rt.mat <- rtVisual.wide[,3:5] # extract rt measures
rtVisual.wide$rtAvg <- rowMeans(rt.mat) # calculate average
summary(aov(rtAvg~group,data=rtVisual.wide))
```

```
##             Df Sum Sq Mean Sq F value Pr(>F)
## group        1   5348    5348     1.6   0.23
## Residuals   10  33407    3341
```

split-plot designs

within-group parts of ANOVA use error term that is average of error terms in separate 1-way ANOVAs

```
##              Sum Sq num Df Error SS den Df F value  Pr(>F)    
## (Intercept) 9610000      1   100222     10  958.87 2.9e-11 ***
## group         16044      1   100222     10    1.60   0.234    
## angle         28850      2   114311     20    2.52   0.105    
## group:angle   50772      2   114311     20    4.44   0.025 *  
```

```
summary(rt.young)[[2]]
```

```
##             Df Sum Sq Mean Sq F value Pr(>F)   
## angle        2  72078   36039    9.71 0.0045 **
## Residuals   10  37122    3712
```

```
summary(rt.old)[[2]]
```

```
##             Df Sum Sq Mean Sq F value Pr(>F)
## angle        2   7544    3772    0.49   0.63
## Residuals   10  77189    7719
```

SS_{error} = 114311 = 37122 + 77189

MS_{error} = 5716 = (3712+7719)/2

linear contrasts

- on between-subject variable:
  - calculate mean score for each subject
  - apply contrast weights to mean scores as in a 1-way design
- on within-subject variable:
  - use contrast weights to convert measures to composite scores
  - use t-test or anova to determine if scores differ across groups (i.e., contrast x group interaction)

linear contrasts example

test of overall contrast <u>ignoring</u> group differences

```
> y.mat<-as.matrix( myData[,2:4] )
> lin.C <- c(-1,0,1)
> myData$lin.scores <- y.mat %*% lin.C
> myData
   group a1 a2 a3 lin.scores
1  young 50 47 51          1
2  young 41 57 43          2
3  young 42 63 40         -2
4  young 46 66 47          1
5  young 45 61 38         -7
6  young 45 57 53          8
7    old 48 39 38        -10
8    old 55 72 54         -1
9    old 51 44 51          0
10   old 53 65 53          0
11   old 68 58 62         -6
12   old 65 37 55        -10
```

mean contrast (linear trend) does not differ significantly from zero

```
> t.test(myData$lin.scores)

        One Sample t-test
data:  myData$lin.scores
t = -1.301, df = 11, p-value = 0.2199
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 -5.384  1.384
sample estimates:
mean of x
       -2
```

linear contrasts example

test of overall contrast <u>ignoring</u> group differences

```
> library(emmeans)
> # create aov_car object:
> rtAge.aov.10 <- aov_car(rt ~ group*angle + Error(subj/angle),
+                 data=rtVisual)
> linTrendWeights <- c(-1,0,1)
> # linear trend ignoring group
> angle.emm <- emmeans(rtAge.aov.10,specs="angle")
> contrast(angle.emm,method=list(linTrendWeights))
 contrast    estimate   SE df t.ratio p.value
 c(-1, 0, 1)      -20 14.1 10  -1.423  0.1851

Results are averaged over the levels of: group
```

linear contrasts example

evaluating group differences

```
> y.mat<-as.matrix( myData[,2:4] )
> lin.C <- c(-1,0,1)
> myData$lin.scores <- y.mat %*% lin.C
> myData
   group a1 a2 a3 lin.scores
1  young 50 47 51          1
2  young 41 57 43          2
3  young 42 63 40         -2
4  young 46 66 47          1
5  young 45 61 38         -7
6  young 45 57 53          8
7    old 48 39 38        -10
8    old 55 72 54         -1
9    old 51 44 51          0
10   old 53 65 53          0
11   old 68 58 62         -6
12   old 65 37 55        -10
```

contrast (linear trend) does not differ significantly between groups

```
> t.test(lin.scores~group,data=myData)

        Welch Two Sample t-test
data:  lin.scores by group
t = 1.779, df = 9.994, p-value = 0.1056
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -1.263 11.263
sample estimates:
mean in group young   mean in group old
                0.5                -4.5
```


linear contrasts example

evaluating group differences

```
> y.mat<-as.matrix( myData[,2:4] )
> lin.C <- c(-1,0,1)
> myData$lin.scores <- y.mat %*% lin.C
> myData
   group a1 a2 a3 lin.scores
1  young 50 47 51          1
2  young 41 57 43          2
3  young 42 63 40         -2
4  young 46 66 47          1
5  young 45 61 38         -7
6  young 45 57 53          8
7    old 48 39 38        -10
8    old 55 72 54         -1
9    old 51 44 51          0
10   old 53 65 53          0
11   old 68 58 62         -6
12   old 65 37 55        -10
```

contrast (linear trend) does not differ significantly between groups

```
> t.test(lin.scores~group,data=myData)

        Welch Two Sample t-test
data:  lin.scores by group
t = 1.779, df = 9.994, p-value = 0.1056
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -1.263 11.263
sample estimates:
mean in group young   mean in group old
                0.5                -4.5
```

linear contrasts example

evaluating group differences with emmeans

```
> # linear trend for each group
> angle.emm.2 <- emmeans(rtAge.aov.10,specs="angle",by="group")
> contrast(angle.emm.2,method=list(linTrendWeights))
group = young:
 contrast     estimate   SE df t.ratio p.value
 c(-1, 0, 1)         5 19.9 10   0.252  0.8065

group = old:
 contrast     estimate   SE df t.ratio p.value
 c(-1, 0, 1)       -45 19.9 10  -2.264  0.0470

> # linear trend x group interaction
> contrast(angle.emm.2,interaction=c("poly","consec"),by=NULL)
 angle_poly group_consec estimate    SE df t.ratio p.value
 linear     old - young       -50  28.1 10  -1.779  0.1056
 quadratic  old - young       307 143.2 10   2.142  0.0578
```